실수 a와 함수 $f(x)=\ln(x^4+1)-c$ ($c>0$인 상수)에 대하여 함수 $g(x)$를

$$g(x)=\int_a^x f(t)dt \text{ —— } g'(x)=f(x)$$

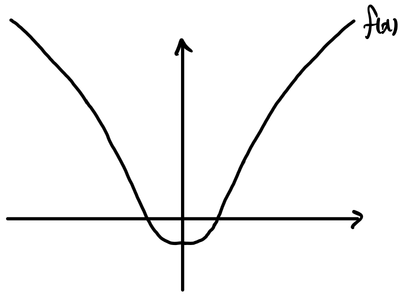
라 하자. 함수 $y=g(x)$의 그래프가 x축과 만나는 서로 다른 점의 개수가 2가 되도록 하는 모든 a의 값을 작은 수부터 크기순으로 나열하면
α_1, α_2, $\cdots$, α_m (m은 자연수)이다.
$a=\alpha_1$일 때, 함수 $g(x)$와 상수 k는 다음 조건을 만족시킨다.

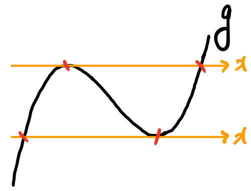
(가) 함수 $g(x)$는 $x=1$에서 극솟값을 갖는다.—— $g'(1)=0$
(나) $\int_{\alpha_1}^{\alpha_m} g(x)dx = k\alpha_m \int_0^1 \lvert f(x) \rvert dx$

$mk\times e^c$의 값을 구하시오. [4점] 16

$f(1)=0$　　$\therefore c=\ln 2$

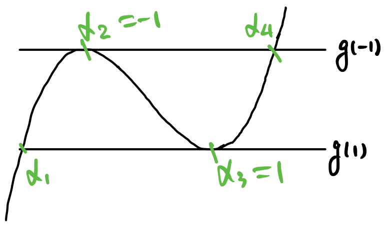
$g'(x)=f(x)=\ln(x^4+1)-\ln 2=0$　$\therefore x=\pm 1$





$\therefore$ a가 될 수 있는 값 : 4개 $\left(\because g(a)=\int_a^a =0\right)$

$\therefore m=4$



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f : 우함수 $\longrightarrow$ g : 점대칭

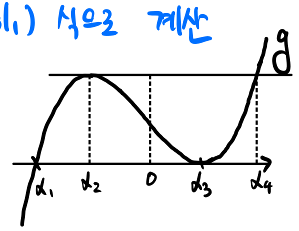
$$\begin{aligned} g(-x) &= \int_a^{-x} f(t)\,dt \quad \big) -t=s \text{ 치환} \\ &= \int_{-a}^{x} f(-s)\cdot(-ds) \\ &= \int_x^{-a} f(s)\,ds \\ &= \int_x^{a} f(s)\,ds + \int_a^{-a} f(s)\,ds \\ &= -g(x) + 2g(0) \end{aligned}$$

$\therefore g(x) + g(-x) = 2g(0)$

$\longrightarrow (0, g(0))$ 대칭

$\int_{\alpha_1}^{\alpha_4} g(x)\,dx$ 를 먼저 구해보자.

sol.1) 식으로 계산



$$\begin{aligned} \int_{\alpha_1}^{\alpha_4} g(x)\,dx &= \int_{\alpha_1}^{-\alpha_1} g(x)\,dx \\ &= \int_{\alpha_1}^{-\alpha_1} \{2g(0) - g(-x)\}\,dx \\ &= 2g(0)\{(-\alpha_1)-\alpha_1\} - \int_{\alpha_1}^{-\alpha_1} g(-x)\,dx \quad \big) x=-t \text{ 치환} \\ &= 4\alpha_1 g(0) - \int_{-\alpha_1}^{\alpha_1} g(t)(-dt) \\ &= 4\alpha_1 g(0) - \int_{\alpha_1}^{-\alpha_1} g(t)\,dt \end{aligned}$$

$\therefore \int_{\alpha_1}^{\alpha_4} g(x)dx = 2\alpha_1 g(0) = 2\alpha_4 g(0)$

$= 2\alpha_4\{g(0)-g(1)\}$ $(\because g(1)=0)$

$= 2\alpha_4\left\{\int_{\alpha_1}^{0} f(x)dx - \int_{\alpha_1}^{1} f(x)dx\right\}$

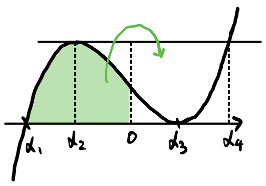
$= 2\alpha_4 \int_{1}^{0} f(x)dx$

$= 2\alpha_4 \int_{0}^{1} -f(x)dx$

$= 2\alpha_4 \int_{0}^{1} |f(x)|dx$ $(\because 0<x<1,\ f(x)<0)$

$\therefore k=2$

sol2) 그래프 특징성 이용



$\therefore \int_{\alpha_1}^{\alpha_4} g(x)dx = \alpha_4 \times g(\alpha_2) = \alpha_4 g(\alpha_2)$

$\int_{0}^{1} |f(x)|dx = -\int_{0}^{1} f(x)dx$

$= \int_{\alpha_1}^{0} f(x)dx + \int_{1}^{\alpha_1} f(x)dx$

$= g(0)-g(1)$

$= g(0)$ $(\because g(1)=0)$

$\therefore \alpha_4 g(\alpha_2) = k\cdot\alpha_4\cdot g(0)$

$\alpha_4\cdot 2g(0) = k\cdot \alpha_4 g(0)$

$\therefore k=2$

$\therefore mk\cdot e^c = 16$

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* 참고 : 대칭성

— $x=a$ 선대칭 $f(a-x)=f(a+x)$

— $(a,0)$ 점대칭 $f(a-x)+f(a+x)=0$

— (a,b) 점대칭 $\dfrac{f(a-x)+f(a+x)}{2}=b$